A robust, discrete, fractional order PID controller dedicated to an oriented PV system

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Zusammenfassung

In the paper the proposition a fractional order, robust, discrete PID controller dedicated to minimum-energy control an interval - parameter, oriented PV system is presented. A tuning of robust controller with use of different cost function is also proposed. Results are by an example depicted.

1 An Introduction

An application of fractional order calculus in modeling and control of dynamic systems has been considered by many Authors, for example Podlubny (see [13],[14]) , Das (see [3]), Kaczorek [4], Pan and Das [11] .

In many situations the use of non integer order controller assures the better control performance, that integer order control. This is caused by the fact, that fractional orders of integration and derivative actions are additional tuning parameters of controller. These paramaters make possible very precise fitting the dynamics of the controller to a controlled plant. The use of fractional order controllers was presented for example by Podlubny in [13] or Petras in [12].

In the paper a propostion of use the fractional order, discrete PID controller to control the elevation angle in the moving part of an experimental oriented PV system. The control plant is described with the use of interval transfer function. The use of interval model is determined by the fact, that the PV works all the year outdoor in extremally different atmospheric conditions.

It is is well known, that an important control problem for oriented PV systems is a minimal energy control. Generally, for integer order control this problem has been considered by many Authors for years, classic solutions of it are well known,but the use of fractional order controllers generates a number of new problems, particularly for uncertain-parameter systems.

In the paper the following problems will be discussed:

- An oriented PV system and its interval model,
- A fractional order PID controller and its discrete approximation,
- A digital closed-loop control system,
- Tuning method for the considered controller
- An Example

2 An oriented PV system and its interval model

Let us consider a moving part of an oriented PV system shown in figure 2. The most simple scheme of this plant is a DC motor with gearbox, considered by many Authors, for example Athans and Falb in [1], Petras [12], p. 121). The simplified scheme of it is shown in figure 1.

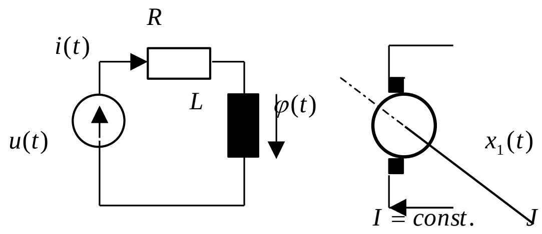

Abbildung 1: A DC electric drive as a model of moving part of the oriented PV system

Abbildung 2: An experimental oriented PV system

The exact description of the plant we deal with can be found in [8], [5], . [6], [7]. Exact parameters of the PV preseted in figure 2 are given in paper [9]. The most simple model of the plant shown in figure 1 has the form of an interval transfer function:

$$G(s,q) = \frac{k}{T_i s} \tag{1}$$

where: $k > 0$ and $T_i > 0$ denote interval parameters of the PV, assembled in vector $q \in Q$ defined as follows:

$$Q = \{q = [k, T_i] : \underline{k} \leq k \geq \overline{k}\ \underline{T_i} \leq T_i \geq \overline{T_i}\} \subset I(R^2) \tag{2}$$

Vertices (corners) of the set Q are defined as underneath:

$$\begin{aligned} q_{ll} &= [\underline{k}, \underline{T_i}] \\ q_{lh} &= [\underline{k}, \overline{T_i}] \\ q_{hh} &= [\overline{k}, \overline{T_i}] \\ q_{hl} &= [\overline{k}, \underline{T_i}] \end{aligned} \tag{3}$$

Vector $q \in Q$ describes parameters of the plant, changing during work of the system outdoor in extremally different atmospheric conditions (summer and winter, with and without snow, etc.). Additionally - these parameters have different values for moving up and moving down the PV. Exemplary values of these parameters are given in the example.

3 A fractional order PID controller and its discrete approximation

A continuous fractional order PID controller is described with the use of the following, continuous, fractional order transfer function:

$$G_c(s,p) = k_P + k_I s^{-\alpha} + k_D s^{\beta} \tag{4}$$

where k_P, k_I and k_D denote coefficients of proportional, integral and derivative actions of the controller, α and β denote fractional orders of the integral and derivative actions. All these parameters can be assembled in a vector p:

$$p = [k_P, k_I, k_D, \alpha, \beta] \tag{5}$$

All the vectors p build the set of permissible controller parameters P, defined as underneath:

$$P = \{p = [k_P, k_I, k_D, \alpha, \beta] : k_P, k_I, k_D > 0, \alpha < 0, \beta > 0\} \subset \mathbf{R}^5 \tag{6}$$

The discrete, fractional order PID controller can be obtained after discretization of time-continuous controller described by (4). The translation can be done with the use of the elementary dependence between continuous and discrete Laplace transforms (see for example [12]):

$$\left(\omega(z^{-1})\right)^{\gamma} = \left(\frac{1+a}{T_s}\right)^{\gamma}\left(\frac{1-z^{-1}}{1+az^{-1}}\right)^{\gamma} = \left(\frac{1+a}{T_s}\right)^{\gamma} CFE\{...\} \tag{7}$$

In (7) a is the coefficient depending on approximation type, T_s denotes the sample time, $CFE...$ is a Continuous Fraction Expansion:

$$CFE_{\gamma}\{\frac{1-z^{-1}}{1+az^{-1}}\} = \frac{v_{\gamma 0} + v_{\gamma 1}z^{-1}}{w_{\gamma 0} + w_{\gamma 1}z^{-1}} \tag{8}$$

Coefficients of discrete transfer function (8) are equal:

$$v_{\gamma 0} = w_{\gamma 0} = \frac{2}{a+\gamma+\gamma a-1}; \quad v_{\gamma 1} = \frac{a-\gamma-\gamma a-1}{a+\gamma+\gamma a-1}; \quad w_{\gamma 1} = 1 \tag{9}$$

In (9) the value of the coefficient a depends on the approximation type, for example, $a = 1$ for Tustin approximation, $a = 0$ for Euler approximation. In further consideration the Euler approximation will be applied. This implies, that coefficients (9) turn to the following simplier form:

$$v_{\gamma 0} = w_{\gamma 0} = \frac{2}{\gamma-1}; \quad v_{\gamma 1} = \frac{-\gamma-1}{\gamma-1}; \quad w_{\gamma 1} = 1 \tag{10}$$

In (10) $\gamma = \alpha$ for integral part of the controller and $\gamma = \beta$ for derivative part respectively. Consequently, the discrete fractional order PID controller can be described with the use of the following discrete transfer function $G_c^+(z^{-1}, p)$, which is also a function of vector p defined by (5):

$$G_c^+(z^{-1}, p) = k_P + k_I\left(\frac{1}{T_s}\right)^{\alpha} CFE_{\alpha} + k_D\left(\frac{1}{T_s}\right)^{\beta} CFE_{\beta} \tag{11}$$

In (11) k_P, k_I, k_D denote gain of proportional, integral and dervative actions of the controller, $\alpha < 0$ denotes the non integer order of integration, $\beta > 0$ denotes the non integer order of the derivation, $CFE_{..}$ is described by (9) and (10). Notice, that the controller (11) can be directly implemented at each digital platform (PLC or microcontroller).

$$G_c^+(z^{-1}, p) = \frac{a_2 + a_1z^{-1} + a_0z^{-2}}{b_2 + b_1z^{-1} + b_0z^{-2}} \tag{12}$$

where:

$$\begin{aligned}
a_0 &= k_P w_{\alpha 1}w_{\beta 1} + k_I v_{\alpha 1}w_{\beta 1} + k_D p_{\beta 1}w_{\alpha 1} \\
a_1 &= k_P(w_{\alpha 0}w_{\beta 1} + w_{\alpha 1}w_{\beta 0}) + k_I(v_{\alpha 0}w_{\beta 1} + v_{\alpha 1}w_{\beta 0}) + k_D(v_{\beta 0}w_{\alpha 1} + v_{\beta 1}w_{\alpha 0}) \\
a_2 &= k_P w_{\alpha 0}w_{\beta 0} + k_I v_{\alpha 0}w_{\beta 0} + k_D v_{\beta 0}w_{\alpha 0}
\end{aligned} \tag{13}$$

$$
\begin{aligned}
b_0 &= w_{\alpha 1} w_{\beta 1} \\
b_1 &= w_{\alpha 0} w_{\beta 1} + w_{\alpha 1} w_{\beta 0} \\
b_2 &= w_{\alpha 0} w_{\beta 0}
\end{aligned} \tag{14}
$$

The whole closed loop control system containing both plant and controller will be described in the next section.

4 The digital closed loop control system

The digital closed loop control system for the plant we deal with is shown in figure 3. The uncertain-parameter plant is described by (1)-(3), the digital fractional order PID controller is described by (11). The problem during tuning the considered control system we deal with is to find such a vector $p_0 \in P$ for which the energy consumption will be minimal or close to minimal in the whole set of uncertain plant parameters Q, defined by (2) and (3). The transfer function of the whole closed-loop control system $G^+_{cl}(z) = \frac{Y^+(z)}{R^+(z)}$

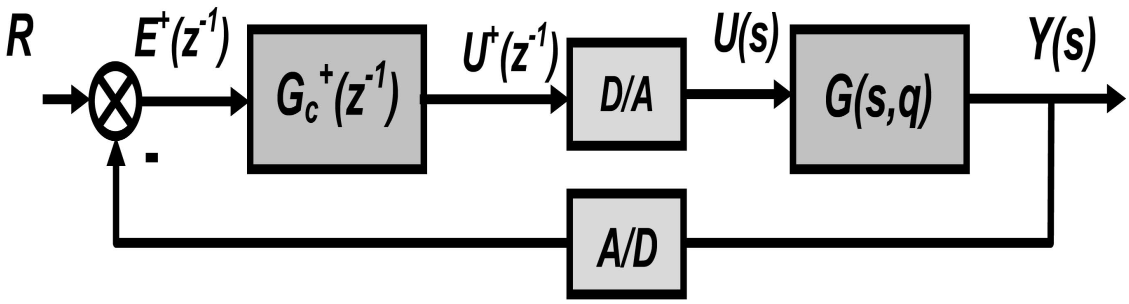

Abbildung 3: A digital closed-loop control system

is described by (15).

$$
G^+_{cl}(z) = \frac{G^+_c(z) G^+(z)}{1 + G^+_c(z) G^+(z)} \tag{15}
$$

where $G^+_c(z^{-1})$ denotes the discrete transfer function of the controller, described with the use of (11)-(14) and $G^+(z^{-1})$ denotes the discrete transfer function of the plant with the zero-order hold at the input:

$$
G^+(z^{-1}, p, q) = c \frac{z^{-1}}{1 - z^{-1}} \tag{16}
$$

where:

$$
c = \frac{kT_s}{T_i} \tag{17}
$$

Finally, with respect to (12) and (16) the discrete, closed-loop transfer function (15) is equal:

$$G_{cl}^{+}(z^{-1},p,q) = \frac{c\left(a_2 z^{-1} + a_1 z^{-2} + a_0 z^{-3}\right)}{b_2 + (b_1 - b_2 + ca_0)z^{-1} + (b_0 - b_1 + ca_1)z^{-2} - (b_0 + ca_0)\, z^{-3}} \quad (18)$$

where $a_{\dots}$, $b_{\dots}$ and c are described by (13), (14) and (17) respectively.

Furthermore, the relationship between $'z'$ trasform of control signal $U^{+}(z^{-1})$ and $'z'$ trasform of a reference signal R can be also given. It has the following form:

$$U^{+}(z^{-1}) = \frac{G_c^{+}(z^{-1})}{1 + G_c^{+}(z^{-1})G^{+}(z^{-1})} R \quad (19)$$

After any simple trasformations the equation 19 turns to the following form:

$$U^{+}(z^{-1}) = \frac{a_2 + (a_1 - a_2)z^{-1} + (a_0 - a_1)z^{-2} - a_0 z^{-3}}{b_2 + (b_1 - b_2 + ca_2)z^{-1} + (b_0 - b_1 + ca_1)z^{-2} + (ca_0 - b_0)\, z^{-3}} R \quad (20)$$

Consequently, the discrete control signal, calculated as inverse z trasform from $U^{+}(z^{-1})$ described by (20) is equal:

$$u^{+}(n) = Z^{-1}\left(U^{+}(z^{-1})\right) \quad (21)$$

Notice, that:

- The parameters of the transfer function (18) are interval numbers, because the coefficient c is the interval number (see (1) - (3)),
- The discrete transfer function (18) is the integer order trasfer function, because non integer orders α and β were replaced by integer order approximation CFE.

The both above remarks allow us to test the properties of the considered digital control system with the use of approach dedicated to discrete, interval, integer order systems.

5 Tuning method for the considered controller

The main goal of use the proposed controller is to minimize an energy consumption during moving the PV from initial to final position.
The energy consumption during moving the PV is described by the following cost function:

$$I(p,q) = T_s \sum_{n=n_0}^{N_f} (u^{+})^2(n) \quad (22)$$

where T_s denotes the sample time, $u^{+}(n)$ denotes the discrete control signal described by (21), n_0 and N_f denote the initial and final time moments of moving the PV system.
In the considered case the problem of optimal tuning the considered fractional order, discrete, robust PID controller consists in finding such a vector $p_0 \in P$ (where P denotes

the set of permissible controller parameters described by (6) which keeps the cost function (22) minimal or close to minimal in the whole set of uncertain plant parameters Q. The vector p_0 can be found with the use of the following algorithm:

1. We calculate vectors p minimizg the cost function (22) for each vertex of set Q separately. Denote these vectors by $p_{0_{ll}}$, $p_{0_{lh}}$, $p_{0_{hl}}$, $p_{0_{hh}}$ respectively. These vectors can be caclulated with the use of MATLAB.

2. We calculate values of cost function (22) for each vertex vector $p_{..}$ calculated in step 1. Denote these values as $I(p_{..}, q_{..})$ respectively. It is easy to notice, that total number of combinations is equal 16. We collect all values of $I(p_{..}, q_{..})$ in a table: rows of the table are associated to vertices $q_{..}$ and columns are associated to vectors $p_{0_{..}}$

3. Finally, as the vector p_0 we select such a vector $p_{..}$, which minimizes one of the following, additional cost functions:

   (a) The average minimal energy consumption:

$$I_{av}(p, q) = 0.25 \sum_q I(p_{..}, q), \quad q \in \{q_{ll}, q_{lh}, q_{hl}, q_{hh}\} \tag{23}$$

   (b) The maximal robustness of control system:

$$I_r(p, q) = |\max(I(p_{..}, q)) - \min(I(p_{..}, q)| \quad q \in \{q_{ll}, q_{lh}, q_{hl}, q_{hh}\} \tag{24}$$

   (c) The minimum from maximal energy consumption:

$$I_{max}(p, q) = \min \max_q I(p_{..}, q), \quad q \in \{q_{ll}, q_{lh}, q_{hl}, q_{hh}\} \tag{25}$$

Selection of certain criterion depends on particular situation during control. The use of the proposed will be shown in the next section.

6 An Example

As an example let us consider the control system described above. We deal with the control of the elevation angle for experimental, oriented PV system shown in figures 1 and 2. The interval parameters of the control plant are given in the table 1. The identification method for these parameters was exactly discussed in paper [9].

Parameter	**Value**
k	$[0.55; 0.64]$
T_i	$[0.57; 0.71]$

Tabelle 1: Interval parameters of experimental PV system

During simulations the sample time in the system was equal 1[s]. The parameters of robust controller were calculated with the use of the algorithm proposed in the previous

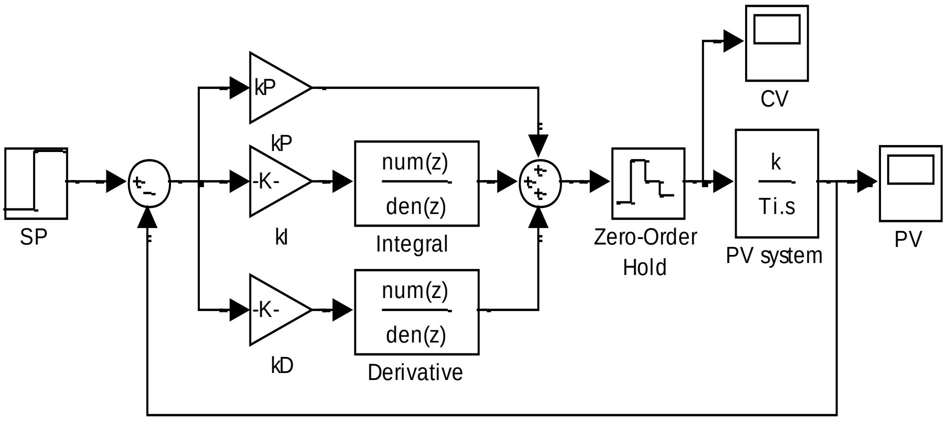

Abbildung 4: The SIMULINK model of the considered closed-loop control system

section. The 1'st step of algorithm (The values of vectors $p_{0_{..}}$ and suitable values of cost function 22) are presented in the table 2 and marked in bold.
The results associated to the 2'nd step of algorithm are also presented in the table 2.

vectors $p_{..}$, $q_{..}$, **Cost function (22)**	$p_{0_{ll}}$= [0.85,0.03,0.05,-0.5,.05];	$p_{0_{lh}}$=[0.8,.03,.05,-0.45,0.21];	$p_{0_{hl}}$=[0.7,0.02,0.05,-0.5,0.3]	$p_{0_{hh}}$=[0.7,0.02,0.05,-0.4,0.9]
q_{ll}=[0.55;0.57]	**0.8767**	0.7934	0.6335	0.6247
q_{lh}=[0.55;0.71]	0.9455	**0.8644**	0.7066	0.6915
q_{hl}=[0.64;0.57]	0.8657	0.7746	**0.6032**	0.5995
q_{hh}=[0.64;0.71]	0.8921	0.8104	0.6527	**0.6418**

Tabelle 2: The 1'st and 2'nd steps of the algorithm

Next the vectors of controller parameters p_0 optimal in the sense of cost functions (23), (24) and (25) can be find with the use of table 2. It is easy to see, that:

1. The minimum of cost function (23) describing the minimum average energy consumption is achieved for vector $p_{0_{hh}}$, the cost function is equal: $I_av(p_{0_{hh}}, q)$=0.6394.

2. The maximal robustness of the control system, described by cost function (24) is achieved for vector $p_{0_{ll}}$, the cost function is equal: $I_r(p_{0_{ll}}, q)$=0.0798.

3. The minimum value of cost function (25) is achieved also for vector $p_{0_{hh}}$, the cost function is equal: $I_max(p_{0_{hh}}, q)$=0.6915.

The exemplary set of step responses of the control system with controller parameters assembled in the vector $p_{0_{hh}}$=[0.7,0.02,0.05,-0.4,0.9] and all vertex vectors describing the plant are shown in figure 5.

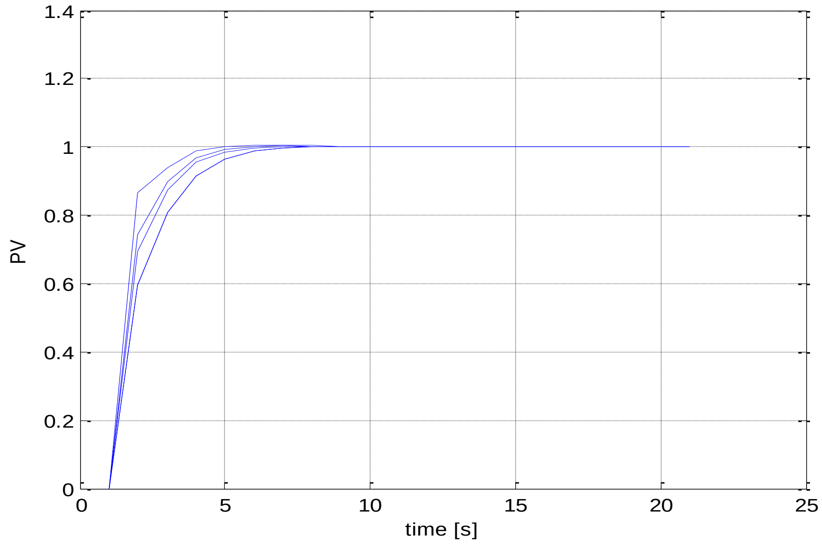

Abbildung 5: The exemplary step responses of control system for all vertices the set Q and vector $p_{0_{hh}}$.

Notice, that the set of controller parameters assures the good control performance in sense another cost functions also: the step response does not have any overshooting and the settling time is resonable.

7 Final conclusions

Final conclusions from the paper can be formulated as follows:

- Results of simulations show, that the proposed robust, fractional order, discrete PID controller assures the good control performance for the considered uncertain parameter oriented PV system.

- The proposed controller can be easily implemented at each digital platform (micro-controller, PLC/PAC). The proposed in this paper fractional order PID controller is recently implemented at SIEMENS PLC, results will be presented soon.

- An another important problem is to propose the analytical method of tuning the proposed PID controller. This also will be considered.

Acknowledgments

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